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NELSON-SIEGEL YIELD CURVE MODEL ESTIMATION AND THE YIELD CURVE TRADING IN THE CROATIAN FINANCIAL MARKET

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Abstract:
The paper examines the possibility of yield curve estimation in the illiquid Croatian financial market using the parametric Nelson-Siegel model. Furthermore bond trading strategies are being discussed regarding the estimated model parameters. Research findings suggest a minimum of 5 data points per observation in order to obtain Nelson-Siegel parameters. The evolution of the parameters seems to be in line with macroeconomic theory considering the state of Croatian economy in the analyzed period. This is especially true in the case of pure kuna instruments sample while some peculiarities have been detected in the sample referring to the euro denominated instruments. However further research should be conducted regarding the applicability of the model for the yield curve trading purposes due to market's illiquidity.

Keywords: yield curve model, bond trading strategies.

Jel Classification: G12

INTRODUCTION

Yield curve models are being used by the central banks and financial market practitioners alike since the information contained in the term structure i.e. the yield curve[2] is important both for conducting business and macroeconomic policy. The economic agents in the emerging financial markets are aware of the importance of the yield curve and its scope of application but are faced with illiquid and undeveloped financial market. This is a straightforward incentive to address the issue. In such an

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[2] Yields on debt securities of equal risk, liquidity and tax characteristics may differ with respect to different remaining time to maturity. A curve connecting the yields in relation to their remaining term to maturity represents the term structure and is called the yield curve.

effort missing and intermittent data due to illiquid and undeveloped financial market presents a special challenge. Also the applicability of the yield curve models in such conditions can be very limited as the available data may not accurately represent market participant's expectations.

Papers related to research in yield curve modeling in illiquid and undeveloped financial markets include works of: Subramanian (2001), Dutta et al. (2005) and Virmani (2006) for Indian, Xie et al. (2006) for Chinese, Chou et al. (2009) for Taiwanese, Smirnov and Zakharov (2003) for Russian, Drenovak (2006) and Zdravkovic (2010) for Serbian, Grum (2006) and Grum and Dolenc (2009) for Slovenian and Aljinovic et al. (2009) and Zoricic (2012) for the Croatian financial market.

In most of the mentioned papers the research is focused on the comparison of different parametric and nonparametric yield curve models in the specific financial market. All of the papers (with the exception of Dutta et al. 2005) include the Nelson-Siegel model in the analysis. Also it should be mentioned that in the case of Slovenian financial market (paper by Grum 2006) the Nelson-Siegel model was pointed out as the preferred model while research in the Serbian market relied on the Nelson-Siegel model exclusively. The Slovenian and Serbian financial markets resemble the Croatian financial market more than any of the other markets mentioned above. Therefore in this paper the possibility of the yield curve estimation in the Croatian financial market using the Nelson-Siegel model is presented. Furthermore the estimation results are used to discuss the yield curve trading opportunities in the market.

METHODOLOGY

The Nelson-Siegel yield curve model falls into a group of parametric yield curve models the most distinctive feature of which is the economic interpretation of their parameters. Based on the estimation of only 4 parameters the Nelson-Siegel is a parsimonious model but it is flexible enough to estimate all of the most shapes a yield curve can take on in practice.

In their paper Nelson and Siegel (1987) begin by defining the forward rate curve in order to attribute to it the desirable smoothness and asymptotic properties. The forward rate curve is defined by the following equation (Nelson and Siegel 1987, 475):

$$f(n,\beta) = \beta_1 + \beta_2 e^{\left(\frac{-n}{\lambda}\right)} + \beta_3 \frac{n}{\lambda} e^{\left(\frac{-n}{\lambda}\right)} \tag{1}$$

where the $f(n,\beta)$ represents the forward yield curve function which depends on maturity n and parameters λ, β_1, β_2 and β_3. Parameter β_1 represents asymptote while parameters ß2 and ß3 enable estimation of various shapes of the yield curve allowed by the model.

By integrating the equation (1) and by dividing by n it is possible to derive the zero coupon yield curve[3] (i.e. the yield curve):

$$Z_n = \beta_1 + (\beta_2 + \beta_3)\left[\frac{1 - e^{-\frac{n}{\lambda}}}{\frac{n}{\lambda}}\right] - \beta_3 e^{\left(-\frac{n}{\lambda}\right)} \tag{2}$$

[3] As the zero coupon yields should represent the average of forward rates.

where Z_n represents the zero coupon yield curve function and β parameters represent yield curve factors.

Because β parameters in the equation (2) can be considered to represent the yield curve factors referring to the level (β_1), slope (β_2) and curvature (β_3) of the zero coupon curve the equation is usually rewritten as (Zdravkovic 2010, 28):

$$Z_n = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \beta_3\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) \tag{3}$$

In such a form the factor loadings can be observed as expressions in the brackets next to β parameters. Figure 1 depicts the evolution of factor loadings in respect to maturity.

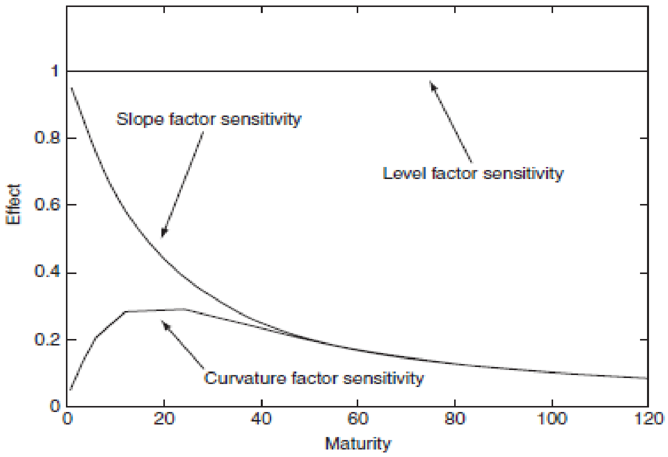

Figure 1. Factor loadings in the Nelson-Siegel model (λ=0,08) (Nyholm 2008)

Furthermore, as shown by Nyholm (2008) the functional form of the Nelson-Siegel model in a regression equation can be represented by:

$$y_t(\tau) = H * \beta_t + e_t, \tag{4}$$

where $y_t(\tau)$ represents a vector of yields observed in the moment t for T maturities in the vector τ, error term is given by $e \sim N(0,R)$ and H represents factor loadings matrix:

$$H = \begin{bmatrix} 1 & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} - \exp(-\lambda\tau_1) \\ 1 & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} - \exp(-\lambda\tau_2) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-\exp(-\lambda\tau_T)}{\lambda\tau_T} & \frac{1-\exp(-\lambda\tau_T)}{\lambda\tau_T} - \exp(-\lambda\tau_T) \end{bmatrix}. \tag{5}$$

Also, following the research by Nyholm (2008) it can be pointed out that parameter λ in the matrix H should be denoted with a time index indicating that it varies with time. However, according to Nyholm (2008) such a dependency increases significantly the number of parameters to estimate while only marginally improving the quality of data fitting by the model. Furthermore it is also possible to treat the λ parameter as a constant rather than as a parameter to estimate thus reducing the functional form of the Nelson-Siegel model to a linear regression that can be solved for different values of λ. The optimal λ is then chosen as the one that minimizes the sum of squared residuals between the observed and fitted yields (Nyholm 2008, 71-72). This approach has been used to estimate the Nelson-Siegel model parameters in the Croatian market.

DATA

The Croatian government has only issued 15 (coupon bearing) bonds in the Croatian financial market that have been listed on the Zagreb Stock Exchange since 2001. Moreover 7 bond issues represent pure domestic currency (kuna) instruments while the remaining 8 were issued with a foreign (euro) currency clause representing de facto euro denominated government bonds in the domestic market. Therefore the yield curve has to be estimated separately for the two samples of the government securities.

The market is not just undeveloped but also very illiquid as many bonds are not being traded on a daily basis. Considering a small number of instruments available and their illiquidity monthly data were used[4].

Because the government began issuing the pure kuna bonds regularly only after 2005[5] yields on government treasury bills were added to the sample as they are continuously available since the beginning of 2003. The yields on domestic currency treasury bills refer to the maturities of 3, 6 and 12 months. Even so 4 or more data points per observation in this sample have only been available since April 2006.

The treasury bills data have also been added to the euro denominated instruments sample in order to enable the comparison of the two samples later on. However it should be noted that the treasury bills with the foreign currency clause have only been issued since March 2009 with the single maturity of 12 months. Even so due to more government bond issues being euro denominated in the last decade 4 or more data points in this sample have been available since March 2004.

Data in both samples were collected up to June 2011.

Yields on government bonds were collected using the Bloomberg trading platform. Since the market convention is to quote the yields on the yield to maturity basis zero coupon yields were calculated using MATLAB's "zbtyield" function that performs bootstrapping.

RESEARCH FINDINGS

The research findings are presented in two subsequent sections. The first section refers to the estimation of the Nelson-Siegel model parameters using the MATLAB code carrying out calculations specified by the equation (4) described earlier while in the second the bond trading strategies are discussed based on the estimated data.

[4] Monthly averages were calculated for all collected yields.
[5] Prior to December 2005 only 2 pure domestic currency bonds have been issued.

Nelson-Siegel yield curve estimation

The estimated Nelson-Siegel model parameters for the pure domestic currency instruments sample are presented in the Table 1. The sample contains 63 observations and only in the case of 1 observation (January 2010) standard errors of the model parameter estimates approached infinity rendering those estimates unreliable.

Table 1. Nelson-Siegel parameter estimates (pure domestic currency instruments)

	Year	Month	SSR[a]	β_1	β_2	β_3	λ	SE[b] β_1	SE β_2	SE β_3
1	2006	4	1,9E-06	0,04	-0,02	0,00	0,26	0,00	0,00	0,01
2		5	1,0E-05	0,04	-0,02	0,02	0,10	0,00	0,00	0,01
3		6	1,0E-05	0,05	-0,02	0,02	0,09	0,00	0,00	0,01
4		7	9,2E-07	0,05	-0,03	0,00	0,25	0,00	0,00	0,00
5		8	1,8E-06	0,05	-0,02	0,00	0,18	0,00	0,00	0,00
6		9	3,0E-06	0,05	-0,02	0,02	0,08	0,00	0,00	0,01
7		10	1,9E-06	0,05	-0,02	0,03	0,08	0,00	0,00	0,00
8		11	1,7E-06	0,05	-0,02	0,00	0,20	0,00	0,00	0,00
9		12	1,7E-06	0,05	-0,03	0,00	0,20	0,00	0,00	0,00
10	2007	1	5,3E-07	0,05	-0,02	0,00	0,07	0,00	0,00	0,00
11		2	3,5E-06	0,05	-0,02	0,00	0,05	0,00	0,00	0,01
12		3	1,0E-05	0,04	0,00	0,05	0,02	0,02	0,02	0,03
13		4	2,0E-05	0,06	-0,03	0,00	0,03	0,01	0,01	0,02
14		5	2,9E-05	0,05	-0,02	0,00	0,05	0,00	0,00	0,02
15		6	9,8E-05	0,06	-0,03	0,00	0,10	0,00	0,01	0,03
16		7	6,2E-05	0,05	-0,03	0,00	0,26	0,00	0,01	0,02
17		8	5,3E-05	0,05	-0,05	0,02	0,26	0,00	0,04	0,06
18		9	5,5E-05	0,06	-0,02	0,00	0,10	0,00	0,00	0,02
19		10	7,0E-05	0,06	-0,02	0,00	0,05	0,01	0,01	0,03
20		11	5,2E-04	0,05	-0,04	0,04	0,26	0,01	0,03	0,08
21		12	3,4E-04	0,05	-0,03	0,02	0,26	0,01	0,03	0,06
22	2008	1	4,1E-04	0,05	-0,01	0,05	0,04	0,03	0,03	0,08
23		2	2,4E-04	0,06	-0,01	0,03	0,06	0,01	0,01	0,05
24		3	2,7E-04	0,06	0,00	-0,05	0,26	0,01	0,01	0,04
25		4	3,3E-05	0,06	-0,02	0,03	0,09	0,00	0,00	0,01
26		5	1,2E-05	0,06	-0,03	0,03	0,26	0,00	0,01	0,01
27		6	1,2E-05	0,06	-0,03	0,02	0,26	0,00	0,01	0,01
28		7	9,5E-06	0,06	-0,03	0,03	0,26	0,00	0,01	0,01
29		8	2,3E-06	0,06	-0,02	0,00	0,26	0,00	0,00	0,01
30		9	7,4E-06	0,06	-0,02	0,02	0,11	0,00	0,00	0,01
31		10	9,6E-06	0,06	-0,01	0,05	0,14	0,00	0,00	0,01
32		11	1,2E-06	0,06	-0,31	0,43	0,22	0,00	0,05	0,06
33		12	1,4E-05	0,06	-0,06	0,16	0,26	0,00	0,02	0,03
34	2009	1	1,5E-05	0,07	-0,01	0,07	0,13	0,00	0,00	0,01
35		2	3,5E-05	0,06	0,00	0,09	0,06	0,01	0,01	0,03
36		3	4,7E-05	0,05	0,02	0,09	0,04	0,02	0,01	0,04
37		4	1,4E-04	0,06	0,01	0,07	0,06	0,02	0,01	0,06
38		5	1,8E-04	0,04	0,04	0,10	0,03	0,08	0,07	0,17
39		6	1,7E-04	0,06	0,02	0,07	0,04	0,03	0,02	0,08
40		7	1,3E-04	0,09	0,16	-0,24	0,26	0,01	0,18	0,26
41		8	1,1E-04	0,09	0,14	-0,22	0,26	0,01	0,11	0,18
42		9	1,0E-05	0,08	0,02	-0,05	0,26	0,00	0,02	0,04
43		10	3,3E-05	0,08	-0,01	-0,01	0,26	0,00	0,01	0,02
44		11	3,1E-04	0,06	-0,03	0,05	0,26	0,01	0,03	0,07
45		12	1,8E-04	0,06	-0,05	0,06	0,26	0,01	0,02	0,05

Table 1. (*continued*)

	Year	Month	SSR[a]	$ß_1$	$ß_2$	$ß_3$	λ	SE[b] $ß_1$	SE $ß_2$	SE $ß_2$
46	2010	1	5,6E-10	0,06	-0,05	0,04	0,06	-	-	-
47		2	3,1E-05	0,06	-0,04	0,00	0,08	0,00	0,00	0,02
48		3	1,4E-05	0,07	-0,05	0,00	0,08	0,00	0,00	0,01
49		4	2,2E-05	0,07	-0,05	0,00	0,10	0,00	0,00	0,02
50		5	1,0E-05	0,07	-0,05	0,00	0,09	0,00	0,00	0,01
51		6	6,7E-05	0,07	-0,05	0,00	0,07	0,01	0,01	0,03
52		7	4,0E-05	0,07	-0,06	0,00	0,20	0,00	0,01	0,02
53		8	3,1E-05	0,07	-0,06	0,00	0,18	0,00	0,01	0,02
54		9	3,0E-05	0,07	-0,06	0,00	0,19	0,00	0,01	0,02
55		10	3,3E-05	0,07	-0,06	0,00	0,21	0,00	0,01	0,02
56		11	2,8E-05	0,07	-0,06	0,00	0,13	0,00	0,00	0,02
57		12	4,0E-05	0,07	-0,06	0,03	0,07	0,00	0,00	0,02
58	2011	1	4,4E-05	0,07	-0,05	-0,01	0,15	0,00	0,01	0,02
59		2	1,7E-05	0,07	-0,05	0,00	0,11	0,00	0,00	0,01
60		3	2,1E-05	0,07	-0,05	0,00	0,07	0,00	0,00	0,01
61		4	1,6E-06	0,05	-0,03	0,07	0,02	0,02	0,01	0,03
62		5	2,5E-05	0,08	-0,07	0,00	0,04	0,01	0,01	0,02
63		6	2,5E-05	0,08	-0,07	0,00	0,05	0,01	0,00	0,02

Source: Zoricic 2012.

[a] Sum of Squared Residuals (SSR)
[b] Standard Error (SE)

The euro denominated instruments sample contains 88 observations but in the case of 11 observations the Nelson-Siegel model parameters could not be reliably estimated as standard errors of the estimates approached infinity. The estimated parameters are presented in the Table 2 (observations in grey refer to the unreliable parameter estimates).

Table 2. Nelson-Siegel parameter estimates (foreign currency clause instruments)

	Year	Month	SSR[a]	$ß_1$	$ß_2$	$ß_3$	λ	SE[b] $ß_1$	SE $ß_2$	SE $ß_3$
1	2004	3	4,4E-07	0,07	0,42	-0,76	0,26	-	-	-
2		4	2,5E-06	0,07	0,32	-0,76	0,26	-	-	-
3		5	2,3E-06	0,07	0,20	-0,60	0,26	-	-	-
4		6	5,9E-08	0,07	0,10	-0,45	0,26	-	-	-
5		7	2,3E-09	0,08	-0,03	-0,04	0,03	-	-	-
6		8	1,1E-08	0,06	92,82	-93,1	0,26	-	-	-
7		9	3,6E-09	0,09	-0,04	-0,09	0,03	-	-	-
8		10	1,9E-10	0,07	0,10	-0,24	0,08	-	-	-
9		11	6,0E-08	0,06	0,15	-0,29	0,09	0,00	0,01	0,01
10		12	1,8E-08	0,06	0,22	-0,33	0,11	0,00	0,01	0,01
11	2005	1	1,1E-08	0,06	0,00	-0,07	0,05	0,00	0,00	0,00
12		2	9,9E-08	0,06	0,02	-0,09	0,08	0,00	0,01	0,01
13		3	1,7E-07	0,06	0,10	-0,20	0,11	0,00	0,02	0,02
14		4	3,5E-09	0,06	0,02	-0,10	0,07	0,00	0,00	0,00
15		5	9,2E-09	0,05	0,01	-0,10	0,08	0,00	0,00	0,00
16		6	1,8E-07	0,05	0,12	-0,22	0,11	0,00	0,01	0,02
17		7	3,6E-07	0,05	0,33	-0,47	0,14	0,00	0,03	0,03
18		8	2,5E-07	0,05	0,13	-0,25	0,12	0,00	0,01	0,01
19		9	2,1E-07	0,05	0,07	-0,16	0,11	0,00	0,01	0,01

Table 2. (*continued*)

	Year	Month	SSR[a]	ß1	ß2	ß3	λ	SE[b] ß1	SE ß2	SE ß3
20		10	3,9E-08	0,05	0,02	-0,09	0,09	0,00	0,00	0,00
21		11	1,0E-06	0,05	2,17	-2,34	0,26	0,00	0,29	0,30
22		12	3,9E-07	0,04	0,60	-0,65	0,26	0,00	0,14	0,14
23	2006	1	4,3E-07	0,04	0,66	-0,74	0,26	0,00	0,11	0,12
24		2	3,9E-07	0,04	0,30	-0,33	0,26	0,00	0,08	0,09
25		3	1,7E-06	0,04	0,20	-0,22	0,26	0,00	0,14	0,15
26		4	5,3E-07	0,05	-0,01	0,01	0,04	0,00	0,00	0,00
27		5	3,2E-07	0,05	-0,01	0,01	0,04	0,00	0,00	0,00
28		6	1,6E-07	0,04	0,01	0,02	0,02	0,00	0,00	0,00
29		7	2,9E-08	0,04	0,00	0,01	0,03	0,00	0,00	0,00
30		8	4,5E-08	0,04	0,00	0,02	0,02	0,00	0,00	0,00
31		9	1,0E-06	0,04	0,01	0,00	0,04	0,00	0,00	0,01
32		10	1,3E-06	0,04	0,00	0,01	0,04	0,00	0,00	0,01
33		11	5,0E-07	0,04	0,00	0,02	0,03	0,00	0,00	0,00
34		12	3,5E-06	0,04	0,04	-0,03	0,26	0,00	0,02	0,03
35	2007	1	5,2E-06	0,05	0,04	-0,03	0,26	0,00	0,01	0,03
36		2	1,2E-06	0,04	0,02	0,00	0,07	0,00	0,00	0,00
37		3	4,6E-06	0,05	0,01	0,00	0,19	0,00	0,00	0,01
38		4	4,7E-06	0,05	0,03	-0,01	0,26	0,00	0,00	0,01
39		5	8,9E-07	0,05	0,01	0,00	0,26	0,00	0,00	0,01
40		6	3,7E-06	0,05	0,00	0,01	0,26	0,00	0,00	0,01
41		7	3,7E-06	0,04	0,01	0,02	0,02	0,02	0,01	0,03
42		8	2,5E-06	0,05	0,01	0,00	0,02	0,01	0,01	0,02
43		9	1,6E-07	0,05	-0,01	0,02	0,26	0,00	0,12	0,13
44		10	2,1E-08	0,06	0,00	-0,01	0,03	0,00	0,00	0,00
45		11	9,6E-08	0,05	0,00	0,01	0,03	0,00	0,00	0,00
46		12	2,2E-07	0,05	-0,35	0,41	0,26	0,00	0,06	0,08
47	2008	1	5,2E-07	0,07	-0,01	-0,04	0,02	0,01	0,01	0,01
48		2	1,2E-06	0,07	-0,01	-0,03	0,02	0,01	0,01	0,02
49		3	1,7E-06	0,05	-0,10	0,14	0,26	0,00	0,08	0,12
50		4	3,7E-06	0,05	-0,07	0,10	0,26	0,00	0,09	0,15
51		5	3,6E-06	0,05	-0,09	0,17	0,26	0,00	0,07	0,12
52		6	4,9E-06	0,05	-0,08	0,15	0,26	0,00	0,06	0,12
53		7	3,6E-06	0,05	-0,03	0,07	0,26	0,00	0,04	0,08
54		8	4,4E-06	0,05	-0,02	0,06	0,26	0,00	0,03	0,08
55		9	9,2E-07	0,06	-0,01	-0,02	0,02	0,01	0,01	0,01
56		10	1,8E-04	0,05	-0,03	0,18	0,26	0,02	0,05	0,35
57		11	8,1E-05	0,06	-0,01	0,03	0,26	0,01	0,01	0,20
58		12	5,5E-06	0,07	0,02	-0,07	0,03	0,01	0,01	0,02
59	2009	1	7,1E-08	0,07	271,5	-271	0,26	-	-	-
60		2	5,9E-11	0,06	0,01	0,04	0,04	-	-	-
61		3	1,3E-06	0,06	0,02	0,00	0,02	0,01	0,01	0,02
62		4	1,4E-06	0,06	0,02	0,00	0,02	0,01	0,01	0,02
63		5	1,5E-06	0,06	0,02	0,00	0,02	0,01	0,01	0,02
64		6	1,6E-06	0,06	0,02	0,00	0,02	0,01	0,01	0,02
65		7	3,2E-06	0,06	-0,04	0,13	0,10	0,00	0,01	0,02
66		8	5,9E-06	0,06	-0,47	0,59	0,26	0,00	0,19	0,23
67		9	4,3E-07	0,08	0,00	-0,06	0,02	0,00	0,00	0,01
68		10	3,5E-08	0,06	0,01	-0,04	0,05	0,00	0,00	0,00
69		11	3,0E-06	0,06	0,00	-0,03	0,07	0,00	0,01	0,02
70		12	1,1E-07	0,03	0,01	0,06	0,02	0,00	0,00	0,01
71	2010	1	1,4E-06	0,08	-0,04	-0,02	0,02	-	-	-
72		2	4,4E-06	0,07	-0,04	0,00	0,04	0,01	0,01	0,02
73		3	7,3E-07	0,06	-0,05	0,00	0,12	0,00	0,00	0,01

Table 2. (*continued*)

	Year	Month	SSR[a]	ß1	ß2	ß3	λ	SE[b] ß1	SE ß2	SE ß3
74		4	4,8E-06	0,06	-0,04	0,00	0,08	0,00	0,01	0,01
75		5	4,5E-06	0,06	-0,05	0,00	0,12	0,00	0,01	0,02
76		6	2,9E-05	0,07	-0,07	0,00	0,14	0,00	0,04	0,06
77		7	3,0E-06	0,07	-0,05	0,00	0,07	0,00	0,00	0,01
78		8	5,4E-06	0,07	-0,05	0,00	0,06	0,00	0,00	0,01
79		9	7,2E-06	0,07	-0,06	0,00	0,08	0,00	0,01	0,02
80		10	2,4E-05	0,02	0,00	0,16	0,02	0,04	0,04	0,08
81		11	1,7E-05	0,01	0,01	0,17	0,02	0,04	0,03	0,07
82		12	3,3E-05	0,07	-0,16	0,04	0,26	0,00	0,23	0,26
83	2011	1	1,3E-05	-0,01	0,03	0,21	0,02	0,03	0,03	0,06
84		2	1,1E-05	-0,01	0,02	0,20	0,02	0,03	0,03	0,05
85		3	3,0E-06	0,07	-0,06	0,00	0,05	0,00	0,00	0,01
86		4	1,1E-06	0,07	0,45	-0,68	0,26	0,00	0,10	0,11
87		5	1,3E-07	0,07	-0,01	-0,12	0,13	0,00	0,00	0,00
88		6	1,7E-06	0,07	0,46	-0,71	0,26	0,00	0,05	0,06

Source: Zoricic 2012.

[a] Sum of Squared Residuals (SSR)
[b] Standard Error (SE)

By comparing the estimated and collected data it was clear that the standard errors of the estimated parameters approached infinity each time the estimation of the Nelson-Siegel model parameters was carried out based on only 4 data points. Therefore this research suggests that using the employed methodology requires at least 5 data points per observation in order to be able to estimate the Nelson-Siegel model parameters.

In order to plot the estimated Nelson-Siegel model parameters referring to the level ($\beta 1$), slope (β_2) and curvature (β_3) of the yield curve both samples were reduced to reliable parameter estimates only. Thus the pure domestic currency instruments sample was reduced by one observation to 62 observations while the foreign currency clause instruments sample was reduced by 11 observations to 77 observations. It should be mentioned that the negative β_2 parameter referring to the slope of the yield curve is related to the normal (upward sloping) yield curve while its positive values are associated with the inverted yield curve.

The evolution of the Nelson-Siegel model parameters for the pure domestic currency securities samples is shown in the Figure 2 below.

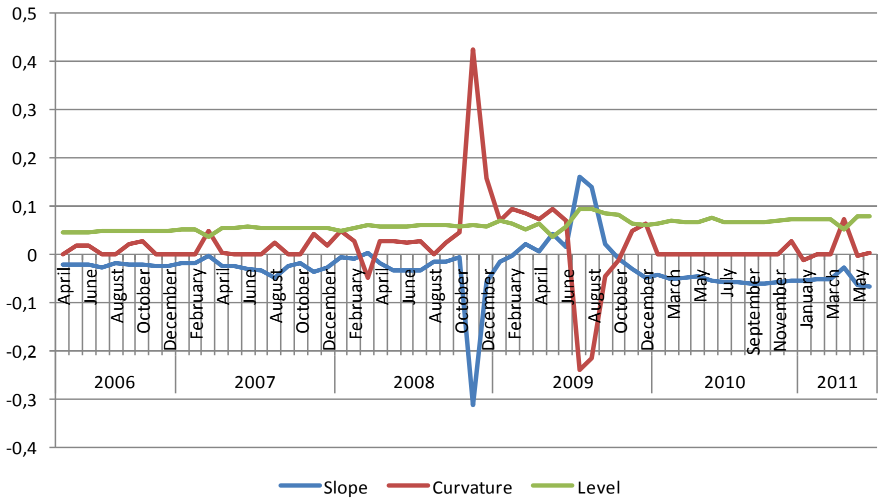

Figure 2. The evolution of the Nelson-Siegel model parameters (Zoricic 2012) (domestic currency instruments sample)

The two spikes in slope and curvature parameters depicted in the graph above reflect the outbreak of the world's economic crisis and the reaction to a change in the fiscal and monetary policies in the Croatian economy. It can be noted that the latter reaction implies inverted yield curves indicating recession ahead which were accompanied by a rise in the level of interest rates. Afterwards the yield curves returned to being upward sloping but much more so than in the beginning of the analyzed period.

The evolution of the Nelson-Siegel model parameters for the foreign currency clause sample is depicted in the Figure 3 below. It is quite obvious that the two analyzed samples are not quite comparable. The problem lies in the fact that the shortest maturity in the euro denominated sample is related to 12 months. Furthermore yields related to the maturity of 12 months have only been continuously available since March 2009. Before that the data related to the short end of the yield curve rested on intermittent data of one government bond approaching maturity. In such conditions the slope and curvature parameters estimates cannot be interpreted as usually due to the fact that the model adjusts itself too much to the given data sample. In order to interpret the evolution of the estimated yield curve estimates of the yields related to the longer maturities should be compared (e.g. two year and ten year[6] yields). Therefore the spikes in the slope and curvature parameters most notably in 2005 and the beginning of 2006 should be considered outliers.

[6] It is not necessary to use yields on longer maturity since the long term yields in the Nelson-Siegel model converge to a certain long term level they cannot exceed.

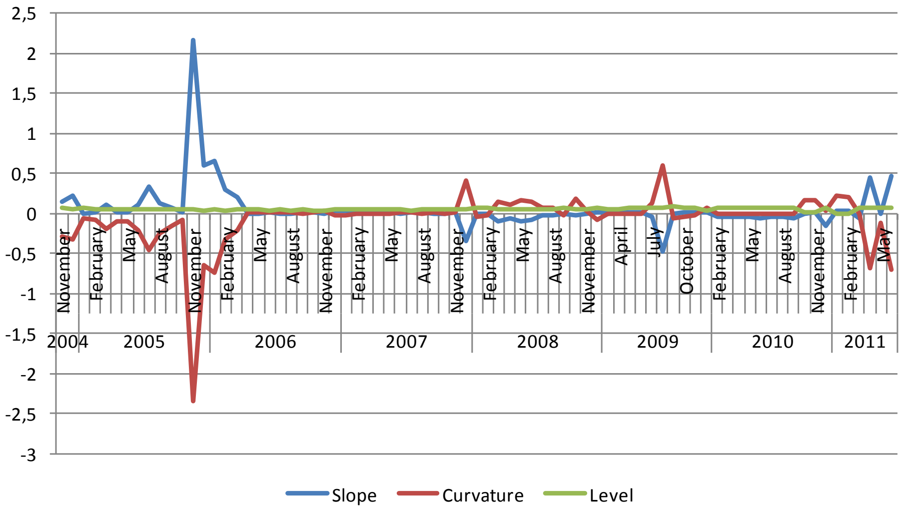

Figure 3. The evolution of the Nelson-Siegel model parameters (Zoricic 2012) (foreign currency clause instruments sample)

However if sharp spikes are omitted the evolution of the parameters resembles the one of the pure domestic currency sample depicted in the Figure 2. There was an upward sloping yield curve present during most of 2006 and 2007. Inverted yield curve appeared even before the world's economic crisis broke out suggesting stronger predictive power of the euro denominated instruments sample. In 2010 the yield curve returns to being upward sloping but with a higher slope than earlier just like in the pure kuna sample.

The following should be pointed out as crucial regarding the conducted analysis:

– It is possible to obtain estimates of the Nelson-Siegel model parameters in the Croatian financial market;
– The evolution of the estimated parameters seems to be in line with macroeconomic theory and the state of the economy in the case of pure kuna sample, while the euro denominated sample seems to exhibit some limitations in this regard due to missing data related to the short end of the yield curve;
– Estimated model parameters can therefore be used to estimate (extrapolate and interpolate) unobservable market yields and to analyze bond trading strategies regarding the shape of the yield curve as will be discussed in the following section of the paper.

Yield curve trading strategies

In order to identify yield curve trading opportunities historical analysis of the yield curve shape must be conducted. Performing such analysis is not always straightforward in the emerging financial markets where the number of outstanding bond issues is limited. In order to consistently analyze yield spreads between maturity segments over time, relevant maturities' yields should be observable in the market. As this is often not the case in emerging markets due to limited number of outstanding issues relevant yields or yield curve shape parameters need to be estimated. As a method of estimating yield curve shape parameters for the purpose of identifying yield curve trading opportunities, the evolution of the estimated Nelson-Siegel model parameters depicted in Figures 2 and 3 will be used. Such approach relies on the Diebold and Li (2006) re-interpretation of the classical term-structure model of Nelson and Siegel (1987). Diebold and Li show that Nelson-Siegel can be used as a modern three-factor yield curve model relying on its level, slope, and curvature parameters to capture the yield curve dynamics. Based on such understanding of time-varying Nelson-Siegel parameters (factors), an estimated series of these factors[7] can be analyzed when appropriate maturity yields are not observable in the market throughout the historical analysis period.

In the absence of relevant market observable yields, identification of spread strategy opportunities should be based on the analysis of the slope parameter (β_2). Since an unusually low (negative) slope parameter would indicate a steep upward sloping yield curve by the model yield curve flattening could be expected. Contrary, unusually high (positive) level of the slope parameter would indicate an inverted curve with a negative slope. A rise in slope (steepening) could therefore be expected. Table 3 summarizes the bond trading strategies that should be executed in line with the above expectations.

Table 3. Strategy identification based on Nelson-Siegel slope parameter (β_2)

β_2	*Instrument position*	
	Short maturity	Long maturity
Low	Short	Long
High	Long	Short

Source: Badurina 2012.

By analogy the curvature parameter (β_3) can be used to identify interest rate term structure curvatures strategies. An unusually high (absolute) value of curvature parameter would indicate that a reduction in the yield curve's curvature can be expected and vice versa. Assuming that a reduction in curvature could be accompanied by flattening and that an increase in curvature could be accompanied by steepening of the yield curve the bond trading strategies that should be executed are summarized in the Table 4.

Table 4. Strategy identification based on Nelson-Siegel curvature parameter (β_3)

β_3	*Instrument position*		
	Short maturity	Middle maturity	Long maturity
Low	Long	Short	Long
High	Short	Long	Short

Source: Badurina 2012.

[7] Diebold and Li (2006) refer to Nelson-Siegel model parameters as "latent factors" since they weren't explicitly interpreted as level, slope and curvature factors in the paper by Nelson and Siegel (1987) when the model was introduced and since they are not directly observable in the market.

Application of the proposed strategy identification rules to estimated Nelson-Siegel slope parameter historical values suggests that yield curve flattening and steepening should have been expected in the late 2008 and mid 2009 respectively for the pure domestic currency sample (see Figure 2). Yield curve flattening should have been expected in the late 2008 also in the euro denominated sample (see Figure 3).

When analyzing historical values of estimated Nelson-Siegel curvature parameter a highly negative value in 2009 can easily be spotted in the pure domestic currency sample. Associated with the appearance of the inverted yield curve such a situation would call for execution of butterfly trade with short position in middle maturity bond in order to capitalize on the increase in term structure curvature. In the foreign currency clause sample positive curvature values can be identified in the mid 2008 accompanied with upward sloping yield curve. The proposed strategy in Table 4 would call for execution of long position in middle maturity instrument and short positions in short and long maturity instruments in such a scenario.

CONCLUSION

The research shows that it is possible to estimate the Nelson-Siegel yield curve model parameters in the illiquid and undeveloped Croatian financial market. The research findings suggest a minimum of 5 data points per observation is required in order to estimate the model parameters. Furthermore the evolution of the estimated parameters seems to be in line with macroeconomic theory and the state of the economy but there are some limitations in this regard when analyzing the foreign currency clause instruments sample. Since there are no continuously available data related to the short end of the yield curve in that sample the resulting estimates of the model parameters exhibit spikes on some occasions which can be considered outliers. Apart from that it can be concluded that the model estimates provide useful information about market participant's expectations which otherwise would not be observable in the market. Such an example is provided by discussing the yield curve trading opportunities based on the estimated Nelson-Siegel model parameters.

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